

Advanced 11+ Multi-Step Problem Solving & Spatial Reasoning

Topic: Probability, Circles, Venn Diagrams & Reverse Percentages | Time Allowed: 45 Minutes | Total Marks: 35

Candidate Name: _____ Date: _____ Score: _____

1. CORE MATHEMATICAL CONCEPTS & RULES

- Probability: $P(\text{Event}) = \frac{\text{Number of successful outcomes}}{\text{Total possible outcomes}}$. $P(\text{Not Event}) = 1 - P(\text{Event})$.
- Reverse Percentages: To find the original quantity before an increase/decrease: $\text{Original} = \frac{\text{Final Value}}{\text{Multiplier}}$ (e.g. after 20% increase, divide by 1.20).
- Circle Properties: Diameter = $2 \times \text{Radius}$. Circumference = $\pi \times d$ (or $2\pi r$). Area = $\pi \times r^2$ (Take $\pi \approx 3.14$ or $\frac{22}{7}$ where required).
- Venn Diagrams: Total = Set A + Set B - Both + Neither.

2. STEP-BY-STEP EXAMINER WORKED EXAMPLE: 11+ REVERSE PERCENTAGE WITH MULTI-STAGE REDUCTIONS

Problem: In a winter sale, a jacket is reduced by 20%. The sale price is GBP 96. What was the original non-discounted price?

Method & Steps:

1. Original price represents 100%.
2. A 20% reduction leaves 80% of the original value: $80\% = \text{GBP } 96$.
3. Find 1% of the original value: $\text{GBP } 96 \div 80 = \text{GBP } 1.20$.
4. Multiply by 100 to find original: $\text{GBP } 1.20 \times 100 = \text{GBP } 120$.
5. Verification check: $20\% \text{ of } \text{GBP } 120 = \text{GBP } 24$. $\text{GBP } 120 - \text{GBP } 24 = \text{GBP } 96$.

Final Solution: GBP 120

3. 11+ EXAM TECHNIQUE & SPEED CALCULATION STRATEGIES

1. Always show full written working: examiners award partial method marks even if arithmetic slips occur.
2. Estimate your answer before calculating to check if your final magnitude makes realistic sense.
3. Watch units carefully (e.g. converting cm to m, millilitres to litres, hours to minutes).
4. In timed exam conditions, spend ~1 minute per mark and flag difficult problems for the end.
5. Specialist 1-to-1 Maths & Science tuition available: www.themathsandsciencetutor.co.uk

Section A: Practice Exercises & Exam Questions

Show all calculations clearly. Write your final answer in the box. Total: 20 Marks

[FLUENCY]

Question 1: Probability without Replacement [3 marks]

A bag contains 5 red, 4 blue, and 3 yellow marbles. One marble is taken at random and not replaced. A second marble is taken. What is the probability that both marbles are red?

Answer / Working:

[FLUENCY]

Question 2: Circle Area & Circumference [3 marks]

A circular garden pond has a radius of 7 metres. Using $\pi = 22/7$, calculate: (a) the circumference (b) the surface area.

Answer / Working:

[REASONING]

Question 3: Venn Diagram Problem [2 marks]

In a class of 32 pupils: 18 play piano, 15 play violin, and 6 play both instruments. How many pupils play neither instrument? Show working.

Answer / Working:

[REASONING]

Question 4: Reverse Percentage [3 marks]

A tree grows in height by 15% over one year, reaching a new height of 4.60 metres. What was its height at the start of the year?

Answer / Working:

[WORD PROB]

Question 5: Pencils & Notebooks Simultaneous [3 marks]

3 notebooks and 4 pens cost GBP 11.50. 2 notebooks and 3 pens cost GBP 8.00. Find the individual price of one notebook and one pen.

Answer / Working:

[REASONING]

Question 6: Surface Area of Cuboid [2 marks]

A wooden rectangular block measures 6 cm by 4 cm by 5 cm. Calculate the total surface area of all six faces in cm^2 .

Answer / Working:

Section B: 11+ Grammar & Independent School Challenges

Advanced multi-step reasoning problems from top UK selective entrance exams.

CHALLENGE 1: THE SHADED ANNULUS GEOMETRY PROBLEM [3 MARKS]

A circular archery target has an outer circle of radius 10 cm and an inner gold bullseye of radius 4 cm.

Using $\pi \approx 3.14$, calculate the exact area of the outer ring (excluding the bullseye) to 1 decimal place.

Final Answer:

CHALLENGE 2: THE WATER FLOW RATE & TANK DRAINER [3 MARKS]

Pipe A fills a reservoir tank in 6 hours. Pipe B fills the same tank in 4 hours. Drain C empties the full tank in 12 hours.

If all three are opened simultaneously when the tank is empty, how many hours will it take to fill the tank completely?

Final Answer:

ROUGH WORKING / TRIAL & IMPROVEMENT SPACE

Section C: Examiner Mark Scheme & Method Solutions

Step-by-step methods, mark allocations, and alternative approaches.

Question 1 [Fluency]

Answer: 5/33 (or ~15.15%)

Method: Total marbles = 12. $P(1\text{st Red}) = 5/12$. Remaining reds = 4, total remaining = 11. $P(2\text{nd Red}) = 4/11$. $P(\text{Both Red}) = (5/12) \times (4/11) = 20/132 = 5/33$.

Question 2 [Fluency]

Answer: (a) Circumference = 44 m | (b) Area = 154 m²

Method: (a) $C = 2 \times (22/7) \times 7 = 44$ m. (b) $A = (22/7) \times 7^2 = (22/7) \times 49 = 22 \times 7 = 154$ m².

Question 3 [Reasoning]

Answer: 5 pupils play neither

Method: Total playing at least one = $18 + 15 - 6 = 27$. Pupils playing neither = $32 - 27 = 5$.

Question 4 [Reasoning]

Answer: 4.00 metres (4 metres)

Method: Original $\times 1.15 = 4.60$ m. Original = $4.60 \div 1.15 = 4.00$ m.

Question 5 [Word Prob]

Answer: Notebook = GBP 2.50 | Pen = GBP 1.00

Method: Eq 1: $3N + 4P = 11.50$ ($\times 2 \rightarrow 6N + 8P = 23.00$). Eq 2: $2N + 3P = 8.00$ ($\times 3 \rightarrow 6N + 9P = 24.00$). Subtract: $P = \text{GBP } 1.00$. Sub in: $2N + 3 = 8 \rightarrow 2N = 5 \rightarrow N = \text{GBP } 2.50$.

Question 6 [Reasoning]

Answer: 148 cm²

Method: Surface Area = $2(lw + lh + wh) = 2(6 \times 4 + 6 \times 5 + 4 \times 5) = 2(24 + 30 + 20) = 2(74) = 148$ cm².

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