Worksheet 4.4: Half-Life Calculations

PhysicsSheets

Specification: GCSE Physics (AQA 4.4.2.3 / Edexcel SP15)

Core Half-Life Equations:

$\mathbf{A = A_0 \left(\frac{1}{2}\right)^n}$ (Remaining Activity/Count-Rate = Initial Activity/Count-Rate × $(1/2)^{\text{number of half-lives}}$)

$\mathbf{n = \frac{t}{t_{1/2}}}$ (Number of half-lives = Total time / Half-life period)

Rearranged Forms:

$A_0 = \frac{A}{\left(\frac{1}{2}\right)^n}$ | $\left(\frac{1}{2}\right)^n = \frac{A}{A_0}$ | $t = n \times t_{1/2}$ | $t_{1/2} = \frac{t}{n}$

Units:

A (Remaining activity/count-rate) in **Becquerels (Bq)** or **counts/second**

A_0 (Initial activity/count-rate) in **Becquerels (Bq)** or **counts/second**

n (Number of half-lives) is **dimensionless**

t (Total time elapsed) and $t_{1/2}$ (Half-life period) must be in **consistent units** (e.g., both in seconds, both in hours, etc.)

Section A: Formula Recall, Units & Conversions

1. State the correct SI unit and its symbol for each variable in the half-life formula:

a) Activity (A or A_0): **Becquerel (Bq)**

b) Time (t or $t_{1/2}$): **Second (s)**

2. Rearrange the formula $A = A_0 \left(\frac{1}{2}\right)^n$ to make A_0 the subject.

Model Answer:

$A_0 = \frac{A}{\left(\frac{1}{2}\right)^n}$

3. Rearrange the formula $n = \frac{t}{t_{1/2}}$ to make $t_{1/2}$ the subject.

Model Answer:

$t_{1/2} = \frac{t}{n}$

4. Perform the following unit conversions. Show your working where appropriate.

a) Convert 2.5 kBq to Becquerels (Bq).

Model Answer:

2.5 kBq = 2.5 × 1000 Bq = 2500 Bq

b) Convert 30 minutes to seconds.

Model Answer:

30 minutes = 30 × 60 s = 1800 s

c) Convert 0.004 Bq to milliBecquerels (mBq).

Model Answer:

0.004 Bq = 0.004 × 1000 mBq = 4 mBq

d) Convert 2.5 years to days (assume 1 year = 365 days).

Model Answer:

2.5 years = 2.5 × 365 days = 912.5 days

e) Convert 72 hours to days.

Model Answer:
$72 \text{ hours} = \frac{72}{24} \text{ days} = 3 \text{ days}$

Section B: Practice Calculations

5. Answer the following questions, showing all your working clearly.

a) A radioactive isotope has an initial activity of 800 Bq and a half-life of 2 hours. Calculate its activity after 6 hours.

Model Answer:
First, calculate the number of half-lives, n:
$n = \frac{t}{t_{1/2}} = \frac{6 \text{ hours}}{2 \text{ hours}} = 3$
Then, calculate the remaining activity, A:
$A = A_0 \left(\frac{1}{2}\right)^n = 800 \text{ Bq} \times \left(\frac{1}{2}\right)^3 = 800 \text{ Bq} \times \frac{1}{8} = 100 \text{ Bq}$

b) A sample of Iodine-131 has an initial activity of 1200 Bq. Its half-life is 8 days. What will its activity be after 24 days?

Model Answer:
First, calculate the number of half-lives, n:
$n = \frac{t}{t_{1/2}} = \frac{24 \text{ days}}{8 \text{ days}} = 3$
Then, calculate the remaining activity, A:
$A = A_0 \left(\frac{1}{2}\right)^n = 1200 \text{ Bq} \times \left(\frac{1}{2}\right)^3 = 1200 \text{ Bq} \times \frac{1}{8} = 150 \text{ Bq}$

c) A radioactive source has an activity of 50 Bq after 40 minutes. If its half-life is 10 minutes, what was its initial activity?

Model Answer:
First, calculate the number of half-lives, n:
$n = \frac{t}{t_{1/2}} = \frac{40 \text{ minutes}}{10 \text{ minutes}} = 4$
Then, rearrange $A = A_0 \left(\frac{1}{2}\right)^n$ to find A_0:
$A_0 = \frac{A}{\left(\frac{1}{2}\right)^n} = \frac{50 \text{ Bq}}{\left(\frac{1}{2}\right)^4} = \frac{50 \text{ Bq}}{\frac{1}{16}} = 50 \text{ Bq} \times 16 = 800 \text{ Bq}$

d) A sample of a radioactive isotope has an initial count-rate of 640 counts/second. After 45 seconds, its count-rate has fallen to 80 counts/second. Calculate the half-life of this isotope.

Model Answer:
First, determine the number of half-lives, n, by repeatedly halving the initial activity until the final activity is reached:
$640 \xrightarrow{1} 320 \xrightarrow{2} 160 \xrightarrow{3} 80$
So, $n = 3$ half-lives.
Then, calculate the half-life, $t_{1/2}$:
$t_{1/2} = \frac{t}{n} = \frac{45 \text{ s}}{3} = 15 \text{ s}$

Section C: Challenge Context & Multi-step Problems

6. A medical tracer containing a radioactive isotope is injected into a patient. The initial activity of the tracer is 2.4 MBq. The half-life of the isotope is 6 hours.

a) Calculate the activity of the tracer remaining in the patient after 18 hours. Give your answer in Bq to an appropriate number of significant figures.

Model Answer:

First, convert initial activity to Bq: $A_0 = 2.4$ MBq $= 2.4 \times 10^6$ Bq.

Calculate the number of half-lives, n: $n = \dfrac{t}{t_{1/2}} = \dfrac{18 \text{ hours}}{6 \text{ hours}} = 3$.

Calculate the remaining activity, A: $A = A_0 \left(\frac{1}{2}\right)^n = 2.4 \times 10^6 \text{ Bq} \times \left(\frac{1}{2}\right)^3 = 2.4 \times 10^6 \text{ Bq} \times \frac{1}{8} = 300000$ Bq.
(Or 3.0×10^5 Bq to 2 significant figures, matching the input 2.4 MBq)

b) The medical procedure requires the tracer's activity to fall below 150 kBq before the patient can be discharged. Calculate the minimum total time, in hours, the patient must wait before discharge. Explain why choosing an isotope with a very short half-life for medical tracers is important.

Model Answer:

Initial activity $A_0 = 2.4$ MBq $= 2400$ kBq. Target activity $A = 150$ kBq.
We need to find n such that $A_0 \left(\frac{1}{2}\right)^n \leq A$.
$2400 \xrightarrow{1} 1200 \xrightarrow{2} 600 \xrightarrow{3} 300 \xrightarrow{4} 150$. So, $n = 4$ half-lives.
Total time $t = n \times t_{1/2} = 4 \times 6$ hours $= 24$ hours.

A very short half-life is important for medical tracers because it means the radioactive material decays quickly within the patient's body. This minimises the patient's exposure to harmful ionising radiation, reducing the risk of cell damage or cancer, while still allowing enough time for the diagnostic procedure to be completed.